NUMERICAL ANALYSIS I

The Bisection Method

January 25, 2023

Instructions:

- *When you have finished the exercises, prepare the command line and show the associated programs to your professor who will record that you have finished the work.*
- *The exercises must be carried out under Matlab or Gnu Octave (https://octave.org/download).*
- *The work must be individual.* ***You must bring your personal computer fully charged.***
- ***It is your responsibility to install Matlab or Gnu Octave there.*** *Octave is a free open-source alternative to MATLAB; It works under GNU/Linux, macOS and Microsoft Windows.*
- *Create an easily accessible folder that will store all your files from this course. I would suggest calling it "MATH319". In the folder created above, create a first sub-folder and call it "LectureCodes".*
- *Download the MATLAB/OCTAVE script* `bisection_tp1.m` *from the Blackboard. Go to Course Contents, click on "Computer-Based Exercises" then download* `bisection_tp1.m`. *Make sure the file name remains* `bisection_tp1.m` *with no spaces or parentheses. If your machine added extra characters to the file name (e.g.* `bisection_tp1(1).m`*) after saving the file, you must change the name without spaces or parentheses before opening or running the file in MATLAB/OCTAVE.*

Exercise 1

1. Create a real-valued function of a real variable $f(x) = x^2 - 5$.
2. Find the value of $f(0)$, $f(3)$ and $f(10)$.
3. What means '$\mathsf{x} = [0, 3, 6, 10];$' ?
4. Write a vector 'vect' with 4 entries, whose components are the values of $f(x[i])$.
5. Plot the function f(x) within the range [0,10].
6. Use the command 'plot' to plot the function f(x) within the range [0,10] with red points.

Exercise 2

1. Let $a_1 = 0$ and $b_1 = 10$. Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-8} to the solution of $x^2 - 5 = 0$ lying in the interval $[0, 10]$.
2. Complete the following program corresponding to the bisection method.
3. Use the code to compute the zeros of $f(x) = x^2 - 5$ in the interval $[0, 10]$.
4. Provide the approximated solution and the number of iterations needed to obtain a tolerance of 10^{-8}.
5. For each iteration $k > 1$, compute the error ratio: e^k/e^{k-1}. What do you notice?
6. Test the convergence when using different stopping criteria.

```
function [zero,res,niter]=bisection_tp1(fun,a,b,tol,nmax)
% ======
% STUDENT:
% ID:
% MY COMMAND LINE TO RUN THE CODE:
%
%
% ======
% ZERO=BISECTION_TP1(FUN,A,B,TOL,NMAX) tries to find Finds function zeros
% of the continuous function FUN in the interval
% [A,B] using the bisection method. If
% the search fails an error message is displayed.
% FUN is a function handle associated with an anonymous
% function or a Matlab function.
% ZERO=BISECTION(FUN,A,B,TOL,NMAX,P1,P2,...) passes
% parameters P1,P2,... to the function FUN(X,P1,P2,...)
% [ZERO,RES,NITER]=BISECTION(FUN,...) returns the value
% of the residual in ZERO and the iteration number at
% which ZERO was computed.
% ***********************************
% *********************** Main *****
% ***********************************
x  = [a, (a+b)*0.5, b]; %vector
fx = fun(x);           % vector fx
if ... ' statement ' ...
  error([' The sign of the function at ... \n']);
  elseif fx(1) == 0
    ...          return
  elseif fx(3) == 0
    ...          return
end
niter = 0;
I = (b - a)*0.5;
while 'condition' & niter < nmax
   ...
   if fx(1)*fx(2) < 0
       ...
       ...
       ...
       ...
   elseif fx(2)*fx(3) < 0
       ...
       ...
       ...
       ...
   else
       ...
       ...
 end
end
if  (niter==nmax & 'condition' )
fprintf(['....']);
end
zero = ...
...
...
return
```