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Non-existence Results for a Sequential Fractional Differential Problem

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Summary

In this paper, we study a class of sequential fractional differential inequalities involving Caputo fractional derivatives with different orders. The nonexistence of nontrivial global solutions is investigated in a suitable space via the test function technique and some properties of fractional integrals. Our results are supported by numerical examples.

KEYWORDS:
Caputo fractional derivative, Nonexistence of solutions, sequential fractional differential equation, test function method

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1 | INTRODUCTION

We consider the problem

$$\begin{cases} {}^{C}D_0^{\mu}\left({}^{C}D_0^{\upsilon}u(\tau)\right) = f(\tau, u(\tau)), \ \tau > 0, \ \mu \in (q-1, q), \ \upsilon \in (p-1, p), q, p \in \mathbb{N}, \\ u^{(j)}(0) = b_j, \ \left({}^{C}D_0^{\upsilon}u\right)^{(i)}(0) = c_i, b_j, \ c_i \in \mathbb{R}, \end{cases} \tag{1}$$

where $i = 0, 1, ..., q-1$, $j = 0, 1, ..., p-1$, $q = -[-\mu]$, $p = -[-\upsilon]$ and ${}^{C}D_0^{\sigma}$ is the Caputo fractional derivative (CFD) of order $\sigma > 0$ defined below. A nonexistence result of nontrivial global solutions for (1) will be shown under the condition $f(\tau, u(\tau)) \geq \tau^{\delta} |u(\tau)|^m$ for some $m > 1$ and $\delta \in \mathbb{R}$. That is we consider the problem

$$\begin{cases} {}^{C}D_0^{\mu}\left({}^{C}D_0^{\upsilon}u(\tau)\right) \geq \tau^{\delta} |u(\tau)|^m, \ \tau > 0, \ \mu \in (q-1, q), \ \upsilon \in (p-1, p), q, p \in \mathbb{N}, \\ u^{(j)}(0) = b_j, \ \left({}^{C}D_0^{\upsilon}u\right)^{(i)}(0) = c_i. \end{cases} \tag{2}$$

We seek sufficient conditions on the parameters δ, m, and the initial conditions b_j, c_i, $i = 0, 1, ..., q-1$, $j = 0, 1, ..., p-1$, $q = -[-\mu]$, $p = -[-\upsilon]$, so that nontrivial solutions of (2) do not exist globally.

As, ${}^{C}D_0^{\mu}\left({}^{C}D_0^{\upsilon}u\right) \neq {}^{C}D_0^{\mu+\upsilon}u$, $\mu \in (q-1,q)$, $\upsilon \in (p-1,p)$, $q, p \in \mathbb{N}$, we can not generally apply the power rule here. In case $\mu = 1$, $\upsilon = 0$ and $f(\tau, u(\tau)) = \tau^{\delta}[u(\tau)]^m$ in (1) we obtain the problem

$$\begin{cases} u'(\tau) = \tau^{\delta}[u(\tau)]^m, \ \tau > 0, \ m > 1, \\ u(0) = b. \end{cases} \tag{3}$$

The solution of (3) is

$$u(\tau) = \left[b^{1-m} - \frac{m-1}{1+\delta}\tau^{1+\delta}\right]^{1/(1-m)}, \ \tau > 0, \ \delta \neq -1.$$

Notice that the solution blows-up in finite for $m > 1$.

In case $\mu = \upsilon = 1$, and $f(\tau, u(\tau)) = [u(\tau)]^m$ in (1) we obtain the problem

$$\begin{cases} u''(\tau) = [u(\tau)]^m, \\ u(0) = b_1, \ u'(0) = c_1. \end{cases} \tag{4}$$

When $c_1 = \dfrac{\sqrt{2}}{\sqrt{m+1}} b_1^{(m+1)/2}$, $b_1 > 0$, the solution of (4) is

$$u(\tau) = \left(b_1^{(1-m)/2} - \frac{m-1}{\sqrt{2m+2}}\tau\right)^{2/(1-m)}, \tag{5}$$

and it blows up when

$$\tau = \frac{\sqrt{2m+2}}{m-1} b_1^{(1-m)/2}, \ m > 1.$$

In [15], Kassim et al. studied the problem

$$\begin{cases} {}^{C}D_0^{\mu}u(\tau) + {}^{C}D_0^{\upsilon}u(\tau) \geq \tau^{\delta}|u(\tau)|^m, \ \tau > 0 \\ u^{(k)}(0) = b_k, \quad k = 0, 1, ..., q-1, \end{cases} \tag{6}$$

where $m > 1$, $q \geq 1$ is an integer, $q - 1 < \upsilon \leq \mu < q$, and $b_k \geq 0$, $k = 0, 1, ..., q-1$. They demonstrated, under sufficient conditions, that the problem (6) has no nontrivial global solutions when $m(1-\upsilon) - 1 < \delta < m - 1$.

Recently, Samet showed in [29] that the problem

$$\begin{cases} D_0^{\mu}\left(D_0^{\upsilon}u(\tau)\right) \geq \tau^{\delta}|u(\tau)|^m, \ \tau > 0, \ m > 1, \quad 0 < \upsilon, \mu < 1, \\ \\ I_0^{1-\mu}\left(D_0^{\upsilon}u(0)\right) = b_2, \ I_0^{1-\mu}u(0) = b_1, \end{cases}$$

does not admit nontrivial global solutions when $b_2 > 0$ and $m(1-\mu-\upsilon) - 1 < \delta < m - 1$, where D_0^{σ} is the Riemann–Liouville fractional derivative (RLFD). In this work, we generalize the work of [29] to any value of μ and ν using CFD instead of RLFD. We prove here the nonexistence of nontrivial global solutions, for certain values of δ and m, in an appropriate space which will be specified later. Firstly, we establish some inequalities which will be used in our results. Obviously, sufficient conditions for nonexistence give necessary conditions for existence of solutions. The proof is based on the test function method and some proper manipulations of the fractional derivatives, integrals and the arising terms along the way.

For more results regarding the problem of nonexistence of solutions for fractional differential equations, we refer to [7,10,25,31,17,18,19,24,15,9,14,29,26,1,5,27,3,4] (see also references therein).

In Section 2, we recall some definitions, lemmas, and prepare some material needed to prove our result. Section 3 contains the statement and proof of our nonexistence result illustrated by some examples. Finally in section 4, we present some numerical examples that show the blowing-up character of the solutions.

2 | PRELIMINARIES

In this section, we introduce some definitions, properties, lemmas and notations used in our results. We refer the reader to [16,28,30] for more details concerning fractional derivatives.

Definition 1. [16] Let $AC[0,\infty)$ denote the space of absolutely continuous function on $[0,\infty)$ and $AC^{q}[0,\infty)$, $q\in\mathbb{N}$, denote the space of functions f which have continuous derivatives up to order $q-1$ on $[0,\infty)$ such that $f^{(q-1)}\in AC[0,\infty)$, where $f^{(q-1)}$ denotes the derivative of order $q-1$ of f.

Definition 2. [16] We denote by $L_p(a,b)$, $p\geq 1$, the usual spaces of Lebesgue integrable functions on (a,b).

Definition 3. [16] Let $f\in L_1(a,b)$. *The integrals*

$$I_a^{\mu}f(\tau):=\frac{1}{\Gamma(\mu)}\int_a^{\tau}\frac{f(s)}{(\tau-s)^{1-\mu}}ds,\ \ (\tau>a,\ \mu>0), \tag{7}$$

and

$$I_b^{\mu}f(\tau):=\frac{1}{\Gamma(\mu)}\int_{\tau}^{b}\frac{f(s)}{(s-\tau)^{1-\mu}}ds,\ \ (\tau<b,\ \mu>0), \tag{8}$$

are called the RL *left-sided and RL right-sided* fractional integrals of order μ of the function f, respectively. When $\mu=0$, we set $I_a^0 f=I_b^0 f=f$.

Definition 4. [16] The RL left-sided and RL right-sided fractional derivatives of order $\mu\geq 0$, $q=-[-\mu]$, of the function f are defined by

$$D_a^{\mu}f(\tau):=\left(\frac{d}{d\tau}\right)^{q}I_a^{q-\mu}f(\tau),\ \ \tau>a$$

$$D_b^{\mu}f(\tau):=\left(-\frac{d}{d\tau}\right)^{q}I_b^{q-\mu}f(\tau),\ \ \tau<b,$$

respectively, provided that the right sides are defined almost everywhere on $[a,b]$.

Definition 5. [16] Let $f\in AC^{q}[a,\infty)$. The expression

$${}^{C}D_a^{\mu}f(\tau)=I_a^{q-\mu}f^{(q)}(\tau)=\frac{1}{\Gamma(q-\mu)}\int_a^{\tau}\frac{f^{(q)}(s)}{(\tau-s)^{\mu+1-q}}ds\ \ (\tau>a,\ 0<\mu<1), \tag{9}$$

is called left-sided CFD of order μ of f.

Lemma 1. [16] If $\mu\geq 0$ and $\upsilon>0$, then

$$I_b^{\mu}(b-\tau)^{\upsilon-1}=\frac{\Gamma(\upsilon)}{\Gamma(\upsilon+\mu)}(b-\tau)^{\mu+\upsilon-1}\ \ ,\ \ \tau<b,$$

$$D_b^{\mu}(b-\tau)^{\upsilon-1}=\frac{\Gamma(\upsilon)}{\Gamma(\upsilon-\mu)}(b-\tau)^{\upsilon-\mu-1}\ \ ,\ \ \tau<b.$$

Lemma 2. [16] *Let $\mu>0$, $r\geq 1$, $s\geq 1$ and $\frac{1}{r}+\frac{1}{s}\leq 1+\mu$ ($r\neq 1$ and $s\neq 1$ in the case when $\frac{1}{r}+\frac{1}{s}=1+\mu$). If $\Theta_1\in L_r(a,b)$ and $\Theta_2\in L_s(a,b)$, then*

$$\int_a^b\Theta_1(\tau)\left(I_a^{\mu}\Theta_2\right)(\tau)\,d\tau=\int_a^b\Theta_2(\tau)\left(I_b^{\mu}\Theta_1\right)(\tau)\,d\tau. \tag{10}$$

Lemma 3. [2] Let $\mu>0$ and $q=-[-\mu]$. If f, $I_b^{q-\mu}g\in AC^{q}[a,b]$, then

$$\int_a^b g(\tau)\,{}^{C}D_a^{\mu}f(\tau)\,d\tau=\int_a^b f(\tau)\,D_b^{\mu}g(\tau)\,d\tau+\sum_{i=0}^{q-1}\left[f^{(i)}(\tau)\,D_b^{\mu-i-1}g(\tau)\right]_{\tau=a}^{b}.$$

For $\mathcal{T}>0$ we define the following test function

$$\Theta(\tau)=\begin{cases}\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{\rho},\ 0\leq\tau\leq\mathcal{T},\ \rho>0,\\[2mm] 0,\ \ \tau>\mathcal{T}.\end{cases} \tag{11}$$

Lemma 4. Let $\mu>0$, $q=-[-\mu]$ and Θ be as in (11) with $\rho>\mu-1$. Then $I_{\mathcal{T}}^{q-\mu}\Theta\in AC^{q}\left[0,\mathcal{T}\right]$.

Proof. By using Lemma 1, we find

$$I_{\mathcal{T}}^{q-\mu}\Theta(\tau)=\mathcal{T}^{-\rho}I_{\mathcal{T}}^{q-\mu}(\mathcal{T}-\tau)^{\rho}=\mathcal{T}^{-\rho}\frac{\Gamma(\rho+1)}{\Gamma(\rho+1+q-\mu)}(\mathcal{T}-\tau)^{\rho+q-\mu},\ \tau<\mathcal{T}.$$

Since $\rho>\mu-1$, then $I_{\mathcal{T}}^{q-\mu}\Theta\in AC^{q}\left[0,\mathcal{T}\right]$. □

Lemma 5. Let $\mu, \upsilon > 0$, $p = -[-\upsilon]$ and Θ be as in (11) with $\rho > \mu + \upsilon - 1$. Then $I_{\mathcal{T}}^{p-\upsilon} D_{\mathcal{T}}^{\mu} \Theta(\tau) \in AC^p\left[0, \mathcal{T}\right]$.

Proof. By virtue of Lemma 1, we can write

$$D_{\mathcal{T}}^{\mu} \Theta(\tau) = \mathcal{T}^{-\rho} D_{\mathcal{T}}^{\mu} (\mathcal{T} - \tau)^{\rho} = \mathcal{T}^{-\rho} \frac{\Gamma(\rho+1)}{\Gamma(\rho+1-\mu)} (\mathcal{T} - \tau)^{\rho-\mu},$$

and

$$\begin{aligned} I_{\mathcal{T}}^{p-\upsilon} D_{\mathcal{T}}^{\mu} \Theta(\tau) &= \mathcal{T}^{-\rho} \frac{\Gamma(\rho+1)}{\Gamma(\rho+1-\mu)} I_{\mathcal{T}}^{p-\upsilon} (\mathcal{T} - \tau)^{\rho-\mu} \\ &= \mathcal{T}^{-\rho} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+1+q-\upsilon)} (\mathcal{T} - \tau)^{\rho+p-\mu-\upsilon}, \ \tau < \mathcal{T}. \end{aligned}$$

Since $\rho > \mu + \upsilon - 1$, then $I_{\mathcal{T}}^{p-\upsilon} D_{\mathcal{T}}^{\mu} \Theta(\tau) \in AC^p\left[0, \mathcal{T}\right]$. □

Definition 6. For $\mu \in (q-1, q)$, $\upsilon \in (p-1, p)$, $q, p \in \mathbb{N}$ and $\mathcal{T} > 0$, we introduce the space

$$AC^{\mu,\upsilon}\left[0, \mathcal{T}\right] = \left\{u \in AC^p\left[0, \mathcal{T}\right] : {}^C D_0^{\upsilon} u \in AC^q\left[0, \mathcal{T}\right], \ p \leq q\right\}.$$

Lemma 6. Let $\mu > 0$, $q = -[-\mu]$ and Θ be as in (11) with $\rho > \mu - 1$. If ${}^C D_0^{\upsilon} f \in AC^q\left[0, \mathcal{T}\right]$, $\mathcal{T} > 0$, then

$$\begin{aligned} \int_0^{\mathcal{T}} \Theta(\tau) {}^C D_0^{\mu}\left({}^C D_0^{\upsilon} f(\tau)\right) d\tau &= \int_0^{\mathcal{T}} {}^C D_0^{\upsilon} f(\tau) D_{\mathcal{T}}^{\mu} \Theta(\tau) d\tau \\ &\quad - \sum_{i=0}^{q-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)} \mathcal{T}^{-\mu+i+1} \left({}^C D_0^{\upsilon} f(\tau)\right)^{(i)}(0). \end{aligned}$$

Proof. By Lemma 4, it is clear that $I_{\mathcal{T}}^{q-\mu} \Theta \in AC^q\left[0, \mathcal{T}\right]$. On the other hand, Lemma 1 and (11) imply that

$$D_{\mathcal{T}}^{\mu-i-1} \Theta(\tau) = \mathcal{T}^{-\rho} D_{\mathcal{T}}^{\mu-i-1} (\mathcal{T} - \tau)^{\rho} = \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)} \mathcal{T}^{-\rho} (\mathcal{T} - \tau)^{\rho-\mu+i+1}.$$

Notice that

$$\begin{aligned} D_{\mathcal{T}}^{\mu-i-1} \Theta(0) &= \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)} \mathcal{T}^{-\mu+i+1}, \\ D_{\mathcal{T}}^{\mu-i-1} \Theta(\mathcal{T}) &= 0, \ i = 0, 1, ..., q-1. \end{aligned}$$

By Lemma 3 the result follows. □

Lemma 7. Let $\upsilon, \mu > 0$, $p = -[-\upsilon]$ and Θ be as in (11) with $\rho > \mu + \upsilon - 1$. If $f \in AC^p\left[0, \mathcal{T}\right]$, $\mathcal{T} > 0$, then

$$\begin{aligned} \int_0^{\mathcal{T}} {}^C D_0^{\upsilon} f(\tau) D_{\mathcal{T}}^{\mu} \Theta(\tau) d\tau &= \int_0^{\mathcal{T}} f(\tau) D_{\mathcal{T}}^{\upsilon} \left(D_{\mathcal{T}}^{\mu} \Theta(\tau)\right) d\tau \\ &\quad - \sum_{j=0}^{p-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)} \mathcal{T}^{-\mu-\upsilon+j+1} f^{(j)}(0). \end{aligned}$$

Proof. Lemma 5 tells us $I_{\mathcal{T}}^{p-\upsilon} D_{\mathcal{T}}^{\mu} \Theta(\tau) \in AC^p\left[0, \mathcal{T}\right]$. Moreover, Lemma 1 and (11) yield

$$\begin{aligned} D_{\mathcal{T}}^{\upsilon-j-1} \left(D_{\mathcal{T}}^{\mu} \Theta(\tau)\right) &= \mathcal{T}^{-\rho} D_{\mathcal{T}}^{\upsilon-j-1} \left(D_{\mathcal{T}}^{\mu} (\mathcal{T} - \tau)^{\rho}\right) = \mathcal{T}^{-\rho} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+1)} D_{\mathcal{T}}^{\upsilon-j-1} (\mathcal{T} - \tau)^{\rho-\mu} \\ &= \mathcal{T}^{-\rho} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)} (\mathcal{T} - \tau)^{\rho-\mu-\upsilon+j+1}, \ \tau < \mathcal{T}. \end{aligned}$$

Clearly

$$D_{\mathcal{T}}^{\upsilon-j-1} \left(D_{\mathcal{T}}^{\mu} \Theta\right)(0) = \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)} \mathcal{T}^{-\mu-\upsilon+j+1},$$

and

$$D_{\mathcal{T}}^{\upsilon-j-1} \left(D_{\mathcal{T}}^{\mu} \Theta\right)(\mathcal{T}) = 0, \ j = 0, 1, 2..., p-1.$$

The conclusion in the lemma is a direct application of Lemmas 3 and 5. □

Lemma 8. Let $\mu \in (q-1,q)$, $\upsilon \in (p-1,p)$, $q=-[-\mu]$, $p=-[-\upsilon]$, and Θ be as in (11) with $\rho > \mu+\upsilon-1$. If $f \in AC^{\mu,\upsilon}\left[0,\mathcal{T}\right]$, $\mathcal{T}>0$, then

$$\begin{aligned}\int_0^{\mathcal{T}} \Theta(\tau)^C D_0^{\mu}\left({}^C D_0^{\upsilon} f(\tau)\right) d\tau &= \int_0^{\mathcal{T}} f(\tau)\, D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta(\tau)\right) d\tau \\ &\quad - \sum_{i=0}^{q-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)} \mathcal{T}^{-\mu+i+1}\left({}^C D_0^{\upsilon} f(\tau)\right)^{(i)}(0) \\ &\quad - \sum_{j=0}^{p-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)} \mathcal{T}^{-\mu-\upsilon+j+1} f^{(j)}(0).\end{aligned}$$

Proof. The result follows from Lemmas 6 and 7. □

Lemma 9. Let Θ be as in (11) with $\rho > \mu+\upsilon-1$. Then

$$D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta(\tau)\right) = \mathcal{T}^{-\rho}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}(\mathcal{T}-\tau)^{\rho-\mu-\upsilon},\ \tau<\mathcal{T}.$$

Proof. Thanks to Lemma 1 and (11), we have

$$\begin{aligned}D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta(\tau)\right) &= \mathcal{T}^{-\rho} D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}(\mathcal{T}-\tau)^{\rho}\right) \\ &= \mathcal{T}^{-\rho}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+1)} D_{\mathcal{T}}^{\upsilon}(\mathcal{T}-\tau)^{\rho-\mu} \\ &= \mathcal{T}^{-\rho}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}(\mathcal{T}-\tau)^{\rho-\mu-\upsilon},\ \tau<\mathcal{T}.\end{aligned}$$

□

Lemma 10. Let Θ be as in (11) with $\rho > \upsilon-1$ and $\upsilon, \mu \geq 0$. Then

$$I_{\mathcal{T}}^{\mu}\left|D_{\mathcal{T}}^{\upsilon}\Theta(\tau)\right| = \frac{\Gamma(1+\rho)}{\Gamma(\rho+\mu-\upsilon+1)}\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{\rho+\mu-\upsilon},\ \tau<\mathcal{T}.$$

Proof. Lemma 1 allows us to write

$$\left|D_{\mathcal{T}}^{\upsilon}\Theta(\tau)\right| = \frac{\Gamma(1+\rho)}{\Gamma(\rho-\upsilon+1)}\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{\rho-\upsilon},\ \tau<\mathcal{T},$$

and

$$\begin{aligned}I_{\mathcal{T}}^{\mu}\left|D_{\mathcal{T}}^{\upsilon}\Theta(\tau)\right| &= \frac{\Gamma(\rho+1)}{\Gamma(\rho-\upsilon+1)}\mathcal{T}^{-\rho} I_{\mathcal{T}}^{\mu}(\mathcal{T}-\tau)^{\rho-\upsilon} \\ &= \frac{\Gamma(\rho+1)}{\Gamma(\mu+\rho-\upsilon+1)}\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{\mu+\rho-\upsilon},\ \tau<\mathcal{T}.\end{aligned}$$

□

Lemma 11. Let Θ be as in (11) with $\rho > \max\{p(\upsilon-\mu)-1, \upsilon-1\}$, $\upsilon, \mu \geq 0$ and $p>1$. Then

$$\int_0^{\mathcal{T}} \tau^{\delta(1-p)}\Theta^{1-p}(\tau)\left[I_{\mathcal{T}}^{\mu}\left|D_{\mathcal{T}}^{\upsilon}\Theta\right|(\tau)\right]^p d\tau = C_{\rho,\mu,\upsilon}^{\delta,p}\mathcal{T}^{\delta(1-p)+p(\mu-\upsilon)+1},\ \delta(1-p)+1>0,$$

where

$$C_{\rho,\mu,\upsilon}^{\delta,p} = \left[\frac{\Gamma(\rho+1)}{\Gamma(\mu+\rho-\upsilon+1)}\right]^p \frac{\Gamma(\delta(1-p)+1)\Gamma(p(\mu-\upsilon)+\rho+1)}{\Gamma(\delta(1-p)+p(\mu-\upsilon)+\rho+2)}.$$

Proof. From Lemma 10 and (11), we see that

$$\Theta^{1-p}(\tau)\left[I_{\mathcal{T}}^{\mu}\left|D_{\mathcal{T}}^{\upsilon}\Theta\right|(\tau)\right]^{p} = \left[\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{\rho}\right]^{1-p}\left[\frac{\Gamma(\rho+1)}{\Gamma(\mu+\rho-\upsilon+1)}\right]^{p}\mathcal{T}^{-p\rho}(\mathcal{T}-\tau)^{p(\mu+\rho-\upsilon)}$$
$$= \left[\frac{\Gamma(\rho+1)}{\Gamma(\mu+\rho-\upsilon+1)}\right]^{p}\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{p(\mu-\upsilon)+\rho},\ \tau<\mathcal{T}.$$

Therefore

$$\int_0^{\mathcal{T}}\tau^{\delta(1-p)}\Theta^{1-p}(\tau)\left[I_{\mathcal{T}}^{\mu}\left|D_{\mathcal{T}}^{\upsilon}\Theta\right|(\tau)\right]^{p}d\tau = \left[\frac{\Gamma(\rho+1)}{\Gamma(\mu+\rho-\upsilon+1)}\right]^{p}\mathcal{T}^{-\rho}\times\int_0^{\mathcal{T}}\tau^{\delta(1-p)}(\mathcal{T}-\tau)^{p(\mu-\upsilon)+\rho}\,d\tau.$$

Let $\tau=\mathcal{T}s$. It appears that

$$\int_0^{\mathcal{T}}\tau^{\delta(1-p)}\Theta^{1-p}(\tau)\left[I_{\mathcal{T}}^{\mu}\left|D_{\mathcal{T}}^{\upsilon}\Theta\right|(\tau)\right]^{p}d\tau$$
$$= \left[\frac{\Gamma(\rho+1)}{\Gamma(\mu+\rho-\upsilon+1)}\right]^{p}\mathcal{T}^{\delta(1-p)+p(\mu-\upsilon)+1}\int_0^{1}s^{\delta(1-p)}(1-s)^{p(\mu-\upsilon)+\rho}\,ds$$
$$= \left[\frac{\Gamma(\rho+1)}{\Gamma(\mu+\rho-\upsilon+1)}\right]^{p}\frac{\Gamma(\delta(1-p)+1)\Gamma(p(\mu-\upsilon)+\rho+1)}{\Gamma(\delta(1-p)+p(\mu-\upsilon)+\rho+2)}\mathcal{T}^{\delta(1-p)+p(\mu-\upsilon)+1}.$$

□

Lemma 12. Let Θ be as in (11) with $\rho>p(\mu+\upsilon)-1$, υ, $\mu\geq 0$ and $p>1$. Then

$$\int_0^{\mathcal{T}}\tau^{\delta(1-p)}\Theta^{1-p}(\tau)\left[\left|D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta\right)\right|(\tau)\right]^{p}d\tau = C_{\rho,\mu,\upsilon}^{\delta,p}\mathcal{T}^{\delta(1-p)-p(\mu+\upsilon)+1},\ \delta(1-p)+1>0,$$

where

$$C_{\rho,\mu,\upsilon}^{\delta,p} = \left[\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}\right]^{p}\frac{\Gamma(\rho-p(\mu+\upsilon)+1)\Gamma(\delta(1-p)+1)}{\Gamma(\delta(1-p)+\rho-p(\mu+\upsilon)+2)}$$

Proof. A direct consequence of Lemma 9 is

$$\Theta^{1-p}(\tau)\left[\left|D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta\right)\right|(\tau)\right]^{p} = \left[\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{\rho}\right]^{1-p}\times\left[\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}\right]^{p}\mathcal{T}^{-p\rho}(\mathcal{T}-\tau)^{p(\rho-\mu-\upsilon)}$$
$$= \left[\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}\right]^{p}\mathcal{T}^{-\rho}(\mathcal{T}-\tau)^{\rho-p(\mu+\upsilon)}.$$

Therefore

$$\int_0^{\mathcal{T}}\tau^{\delta(1-p)}\Theta^{1-p}(\tau)\left[\left|D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta\right)\right|(\tau)\right]^{p}d\tau = \left[\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}\right]^{p}\mathcal{T}^{-\rho}\times\int_0^{\mathcal{T}}\tau^{\delta(1-p)}(\mathcal{T}-\tau)^{\rho-p(\mu+\upsilon)}\,d\tau.$$

Next, the change of variable $\tau=s\mathcal{T}$ yields

$$\int_0^{\mathcal{T}}\tau^{\delta(1-p)}\Theta^{1-p}(\tau)\left[\left|D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta\right)\right|(\tau)\right]^{p}d\tau$$

$$= \left[\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}\right]^{p} \mathcal{T}^{\delta(1-p)-p(\mu+\upsilon)+1} \int_{0}^{1} s^{\delta(1-p)} (1-s)^{\rho-p(\mu+\upsilon)} ds$$

$$= \left[\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+1)}\right]^{p} \frac{\Gamma(\delta(1-p)+1)\Gamma(\rho-p(\mu+\upsilon)+1)}{\Gamma(\delta(1-p)+\rho-p(\mu+\upsilon)+2)} \mathcal{T}^{\delta(1-p)-p(\mu+\upsilon)+1}.$$

□

Lemma 13. Let Θ be as in (11) with $\rho > p\upsilon - 1$, $\upsilon > 0$ and $p > 1$. Then

$$\int_{0}^{\mathcal{T}} \tau^{\delta(1-p)} \Theta^{1-p}(\tau) \left| D_{\mathcal{T}}^{\upsilon} \Theta(\tau) \right|^{p} d\tau = C_{\rho,\upsilon}^{\delta,p} \mathcal{T}^{\delta(1-p)-p\upsilon+1}, \ \delta(1-p)+1 > 0,$$

where

$$C_{\rho,\upsilon}^{\delta,p} = \left[\frac{\Gamma(1+\rho)}{\Gamma(\rho-\upsilon+1)}\right]^{p} \frac{\Gamma(\rho-p\upsilon+1)\Gamma(\delta(1-p)+1)}{\Gamma(\delta(1-p)+\rho-p\upsilon+2)}.$$

Proof. This is an immediate consequence of Lemma 1 and a similar change of variable as in the above lemmas. □

Remark 1. If $m, m' > 1$ and $\frac{1}{m} + \frac{1}{m'} = 1$, then $\frac{m'}{m} = m' - 1$, $m' = \frac{m}{m-1}$ and $m(\mu - 1) + 1 > 0 \Longleftrightarrow m'\mu > 1$, for $\mu > 0$.

3 | NONEXISTENCE OF NONTRIVIAL SOLUTIONS

In this section we will demonstrate our result.

Theorem 1. Suppose that

$$q - \mu - (\upsilon + q - 1)m - 1 < \delta < m - 1, \ m > 1, c_{q-1} > 0.$$

Then, Problem (2) has no nontrivial global solution in $AC^{\mu,\upsilon}[0,\infty)$.

Proof. We argue by contradiction. Assume that nontrivial solution u exists for all time $\tau > 0$. Let Θ be as in (11) with $\rho > \frac{m}{m-1}(\mu+\upsilon) - 1$. Multiplying both sides of the inequality in (2) by Θ and integrating over $[0, \mathcal{T}]$, we get

$$I = \int_{0}^{\mathcal{T}} \Theta(\tau) \tau^{\delta} |u(\tau)|^{m} d\tau \leq \int_{0}^{\mathcal{T}} \Theta(\tau) {}^{C}D_{0}^{\mu} \left({}^{C}D_{0}^{\upsilon} u(\tau)\right) d\tau \tag{12}$$

According to Lemma 8, it is obvious that

$$\begin{aligned} \int_{0}^{\mathcal{T}} \Theta(\tau) {}^{C}D_{0}^{\mu} \left({}^{C}D_{0}^{\upsilon} u(\tau)\right) d\tau &= \int_{0}^{\mathcal{T}} u(\tau) D_{\mathcal{T}}^{\upsilon} \left(D_{\mathcal{T}}^{\mu} \Theta(\tau)\right) d\tau \\ &\quad - \sum_{i=0}^{q-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+2+i)} \mathcal{T}^{i+1-\mu} \left({}^{C}D_{0}^{\upsilon} u(\tau)\right)^{(i)}(0) \\ &\quad - \sum_{j=0}^{p-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+2+j)} \mathcal{T}^{-\mu-\upsilon+j+1} u^{(j)}(0) \\ &= \int_{0}^{\mathcal{T}} u(\tau) D_{\mathcal{T}}^{\upsilon} \left(D_{\mathcal{T}}^{\mu} \Theta(\tau)\right) d\tau - \sum_{i=0}^{q-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)} \mathcal{T}^{-\mu+i+1} c_{i} \\ &\quad - \sum_{j=0}^{p-1} \frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)} \mathcal{T}^{-\mu-\upsilon+j+1} b_{j}. \end{aligned} \tag{13}$$

and hence

$$I=\int_0^{\mathcal{T}}\Theta(\tau)\,\tau^{\delta}\,|u(\tau)|^m\le\int_0^{\mathcal{T}}u(\tau)\,D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta(\tau)\right)d\tau-\sum_{i=0}^{q-1}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)}\mathcal{T}^{-\mu+i+1}c_i$$
$$-\sum_{j=0}^{p-1}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)}\mathcal{T}^{-\mu-\upsilon+j+1}b_j,$$

or

$$\begin{aligned}I+\frac{\Gamma(1+\rho)}{\Gamma(\rho-\mu+q+1)}\mathcal{T}^{q-\mu}c_{q-1}\le&\int_0^{\mathcal{T}}u(\tau)\,D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta(\tau)\right)d\tau\\&-\sum_{i=0}^{q-2}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)}\mathcal{T}^{-\mu+i+1}c_i\\&-\sum_{j=0}^{p-1}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)}\mathcal{T}^{-\mu-\upsilon+j+1}b_j.\end{aligned}\tag{14}$$

Next, we insert $\Theta^{1/m}(\tau)\,\tau^{\delta/m}\Theta^{-1/m}(\tau)\,\tau^{-\delta/m}$ inside the integral of (14) and using Young's inequality, we obtain

$$\begin{aligned}I+\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+q+1)}\mathcal{T}^{q-\mu}c_{q-1}\le&\frac{1}{m}\int_0^{\mathcal{T}}\tau^{\delta}\Theta(\tau)\,|u(\tau)|^m\,d\tau\\&+\frac{1}{m'}\int_0^{\mathcal{T}}\Theta^{-m'/m}(\tau)\,\tau^{-\delta m'/m}\left|D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta(\tau)\right)\right|^{m'}d\tau\\&-\sum_{i=0}^{q-2}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)}\mathcal{T}^{-\mu+i+1}c_i\\&-\sum_{j=0}^{p-1}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)}\mathcal{T}^{-\mu-\upsilon+j+1}b_j,\end{aligned}$$

or

$$\begin{aligned}\frac{1}{m'}I+\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+q+1)}\mathcal{T}^{q-\mu}c_{q-1}\le&\frac{1}{m'}\int_0^{\mathcal{T}}\Theta^{-m'/m}\tau^{-\delta m'/m}\left|D_{\mathcal{T}}^{\upsilon}\left(D_{\mathcal{T}}^{\mu}\Theta(\tau)\right)\right|^{m'}d\tau\\-\sum_{i=0}^{q-2}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)}\mathcal{T}^{-\mu+i+1}c_i&-\sum_{j=0}^{p-1}\frac{\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)}\mathcal{T}^{-\mu-\upsilon+j+1}b_j.\end{aligned}\tag{15}$$

The integral term in (15) may be evaluated by Lemma 12

$$\begin{aligned}\frac{m'\Gamma(\rho+1)}{\Gamma(\rho-\mu+q+1)}\mathcal{T}^{q-\mu}c_{q-1}\le&\,C_{\rho,\mu,\upsilon}^{\delta,m'}\mathcal{T}^{\delta\left(1-m'\right)-(\mu+\upsilon)m'+1}\\-\sum_{i=0}^{q-2}\frac{m'\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)}\mathcal{T}^{-\mu+i+1}c_i&-\sum_{j=0}^{p-1}\frac{m'\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)}\mathcal{T}^{-\mu-\upsilon+j+1}b_j.\end{aligned}\tag{16}$$

From (16), we deduce that

$$\begin{aligned}\frac{m'\Gamma(\rho+1)}{\Gamma(\rho-\mu+q+1)}c_{q-1}\le&\,C_{\rho,\mu,\upsilon}^{\delta,m'}\mathcal{T}^{\mu-q+\delta\left(1-m'\right)-(\mu+\upsilon)m'+1}\\-\sum_{i=0}^{q-2}\frac{m'\Gamma(\rho+1)}{\Gamma(\rho-\mu+i+2)}\mathcal{T}^{-q+i+1}c_i&-\sum_{j=0}^{p-1}\frac{m'\Gamma(\rho+1)}{\Gamma(\rho-\mu-\upsilon+j+2)}\mathcal{T}^{-q-\upsilon+j+1}b_j.\end{aligned}$$

If $\delta>q-\mu-(q+\upsilon-1)\,m-1$ we see that $\mu-q+\delta\left(1-m'\right)-(\mu+\upsilon)\,m'+1<0$, and consequently $\mathcal{T}^{\mu-q+\delta\left(1-m'\right)-(\mu+\upsilon)m'+1}$, $\mathcal{T}^{-q+i+1}$, $\mathcal{T}^{-q-\upsilon+j+1}\to0$ as $\mathcal{T}\to\infty$. Therefore

$$c_{q-1}\le0.$$

We reach a contradiction since $c_{q-1}>0$. □

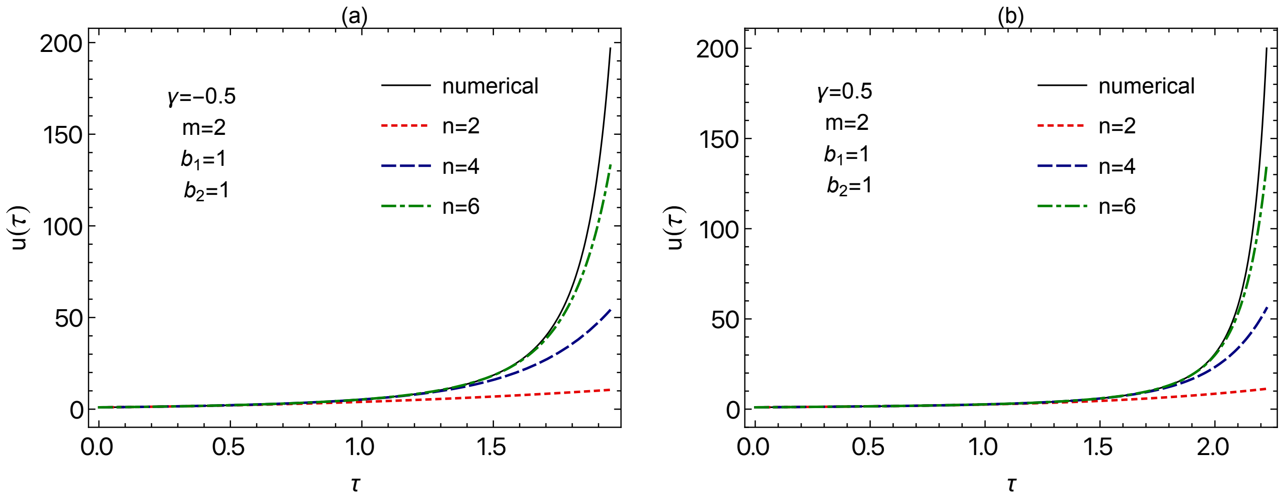

Figure 1 A plot of the numerical solution (black-line) of $u''(\tau) = \tau^{\gamma}|u(\tau)|^m$ is shown and compared to different iterations ($n = 2, 4, 6$) using Eq. (17). γ is taken to be -0.5 and 0.5 in panel (a) and (b), respectively. The initial conditions are taken to be $u(0) = b_1$ and $u'(0) = b_2$, both specified in the figure.

4 | EXAMPLES

The problem (2) is equivalent to the Volterra integral inequality

$$u(\tau) \geq \sum_{k=0}^{p-1} \frac{b_k}{k!}\tau^k + \sum_{k=0}^{q-1} \frac{c_k}{k!}\tau^k + I^{\mu+\upsilon}\left(\tau^{\delta}|u(\tau)|^m\right).$$

To study the behavior of the solution $u(\tau)$ numerically, we will use the iterative scheme where the above inequality can be written as

$$u^{(n)}(\tau) = \sum_{k=0}^{p-1} \frac{b_k}{k!}\tau^k + \sum_{k=0}^{q-1} \frac{c_k}{k!}\tau^k + I^{\mu+\upsilon}\left(\tau^{\delta}\left|u^{(n-1)}(\tau)\right|^m\right), \tag{17}$$

where $n = 1, 2, ...,$ represents the number of iterations starting with the initial guess $u^0 = 1$.

Using Eq. (17), we study some special cases by assuming different μ and ν in every case. We note here that $q = -[-\mu]$, $p = -[-\upsilon]$.

First, let us start by the simplest case when $\mu = \nu = 1$. In this case, the differential equation is ordinary and its solution could be found numerically using any numerical software. In Eq. (5), the analytical solution, which blows-up at finite τ, is presented when $\delta = 0$. In this example, we find the numerical solution of this differential equation using $m = 2$ and $\delta \neq 0$. Then, compare it to the solutions obtained from Eq. (17) after $n = 2, 4, 6$ iterations as shown in Fig. 1. As can be seen from this figure, the agreement to the numerical solution increases by increasing the number of iterations n. This comparison shows that qualitative agreement to the solution could be achieved using a small number of iterations. We will use this argument later when μ and ν are fractions.

Next, we consider solutions of problem (2) using Eq. (17) with different μ and ν fractional orders. In Fig. 2, we plot the solution $u(\tau)$ in three cases: $0 < \mu, \nu < 1$ (top panel), $1 < \mu < 2$ and $0 < \nu < 1$ (middle panel), and $1 < \mu, \nu < 2$ (bottom panel). The number of iterations is taken to be $n = 3$ and $n = 4$ in the left and right columns, respectively. Here we stopped at $n = 4$ due the complexity of the solution at large n and due to the fact that more iterations just result in more powers of τ making the blowing more faster. Even with $n = 4$, the blowing-up character of the solutions is shown at finite τ as can be seen from Fig. 2.

In all cases considered above, the solutions showed a blowing-up character for different parameters. This support the finding of this work where nontrivial global solutions of problem (2) do not exist as discussed in section 3.

In perspective, we aim to extend our theoretical work by exploring other types of fractional differential equations, in addition to conducting numerical analysis and simulations.[8,23,22,21,20].

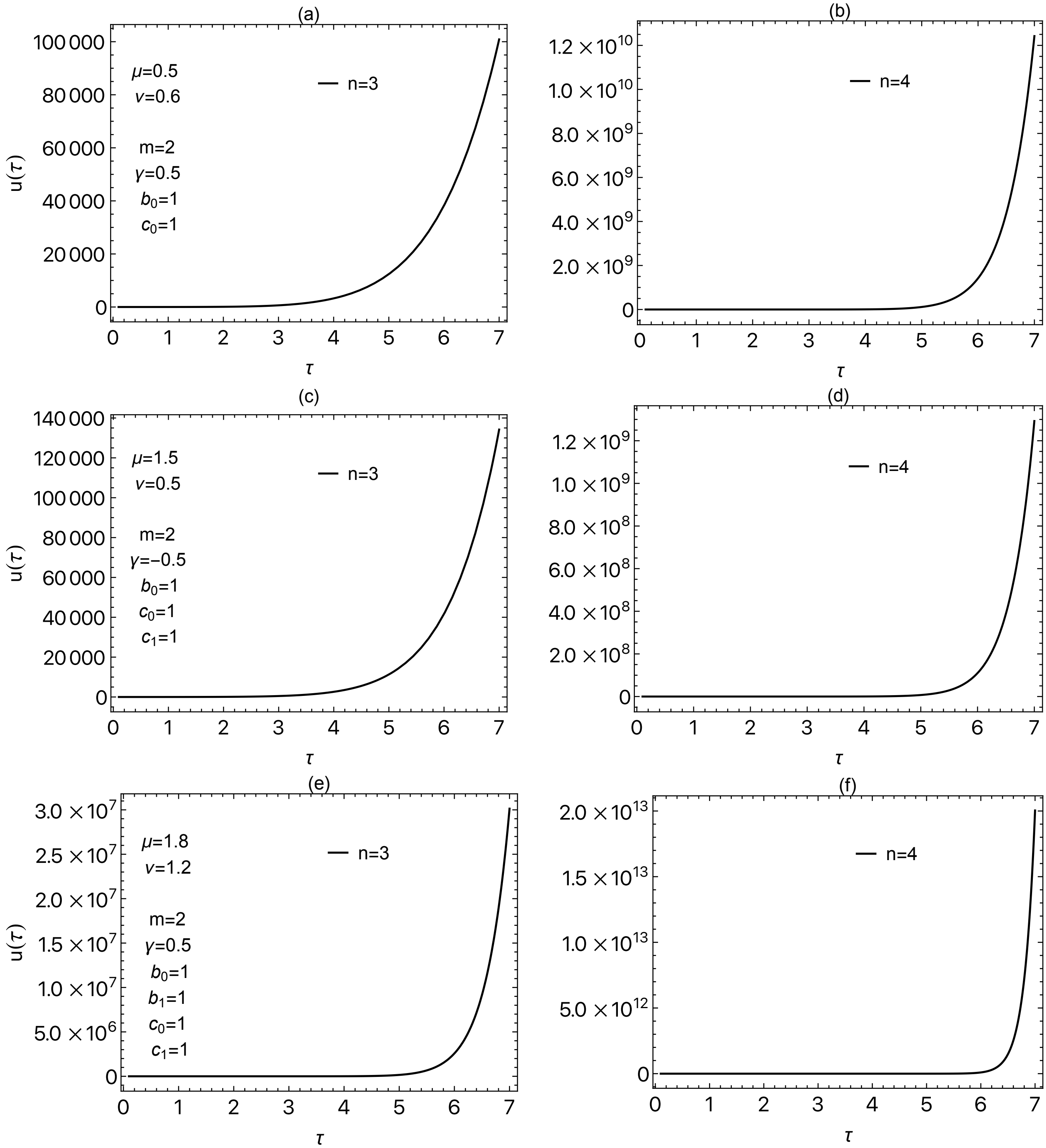

Figure 2 Numerical solutions of Eq. (17) are shown with different μ and ν orders. In the top panel, μ and ν are taken to be between 0 and 1, in the middle panel we consider $1 < \mu < 2$ and $0 < \nu < 1$, and finally in the bottom panel, $1 < \mu, \nu < 2$ are considered. The number of iterations is taken to be $n = 3$ and $n = 4$ in the left and right columns, respectively.

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