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A Framework Combining Optimal Control and Reduced Order Modeling for Accelerated Blood Flow Simulations

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M.Sc. Thesis

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A thesis submitted to Khalifa University of Science and Technology in accordance with the requirements of the degree of Master of Science in Applied Mathematics, Department of Mathematics.



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by

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Abstract

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This thesis presents a computational framework that combines PDE-constrained optimal control and reduced order modeling for accelerated blood flow simulation. The objective is to reconstruct unknown inlet boundary conditions from velocity data available on internal cross-sections of the computational domain. Blood flow is modeled using the stationary incompressible Navier–Stokes equations, and the data assimilation problem is formulated as an optimal control problem with the inlet normal stress acting as the control variable.

The associated optimality system, consisting of the state equations, adjoint equations, and optimality condition, is derived at the continuous level and discretized using a mixed finite element method based on Taylor–Hood elements. A forward–backward sweep strategy combined with a Newton-type update is employed to solve the resulting nonlinear system. Numerical experiments are first conducted for a linear Stokes problem to validate the implementation and to analyze convergence properties. The framework is then extended to the stationary nonlinear Navier–Stokes equations, and a sensitivity study with respect to measurement noise is performed using synthetically generated velocity data.

Since solving the full-order optimality system is computationally demanding, Proper Orthogonal Decomposition (POD) is introduced to construct reduced order models for

both the state and adjoint variables. The full optimality system is projected onto low-dimensional subspaces generated from snapshot data, significantly reducing the computational complexity. For the Stokes case, the reduced optimal control approach achieves speedups of up to one order of magnitude while maintaining relative errors of order 10^{-3} to 10^{-4} .

The results demonstrate that combining optimal control with reduced order modeling provides an efficient and accurate approach for accelerated data assimilation in hemodynamic simulations. Although all numerical experiments are conducted using synthetic data in simplified two-dimensional geometries, the proposed framework establishes a foundation for future extensions to nonlinear, time-dependent, and patient-specific blood flow simulations.

Indexing Terms: Optimal Control, Data Assimilation, Reduced Order Modeling, Proper Orthogonal Decomposition (POD), Navier–Stokes Equations, Finite Element Method

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