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Abstract

This project aims to numerically simulate the deformations of red blood cell (RBC) membranes and to study their stationary shapes. Our approach simplifies the structure of the RBC membrane as a vesicle membrane due to its similarity to a phospholipid bilayer structure. We adopt the well-known Helfrich model, in which the behavior of the membrane is described by minimizing a bending energy depending on the square of the membrane curvature. This results in a highly nonlinear problem, presenting significant challenges in the field of computational fluid dynamics. To solve this problem, we propose a relatively simplified modeling approach using tools from differential geometry. We formulate an optimization problem where the membrane minimizes its bending energy while preserving a fixed surface area and perimeter constraints. This leads to a saddle point problem, where two real Lagrange multipliers are introduced to enforce the constraints. We derive the optimality conditions and solve the resulting initial value problem using various numerical schemes, employing a shooting method.